

A See-Saw Mechanism with light sterile neutrinos

B. H. J. McKellar*University of Melbourne
Parkville, Victoria 3052, Australia

G. J. Stephenson Jr. [†]
University of New Mexico
Albuquerque, New Mexico 87131

T. Goldman[‡]
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

M. Garbutt[§]
University of Melbourne
Parkville, Victoria 3052, Australia

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Abstract

The usual see-saw mechanism for the generation of light neutrino masses is based on the assumption that all of the flavours of right-handed (more properly, sterile) neutrinos are heavy. If the sterile Majorana mass matrix is singular, one or more of the sterile neutrinos will have zero mass before mixing with the active (left-handed) neutrinos and be light after that mixing is introduced. In particular, a rank 1 sterile mass matrix leads naturally to two pseudo-Dirac pairs, one very light active Majorana neutrino and one heavy sterile Majorana neutrino. For any pattern of Dirac masses, there exists a region of parameter space in which the two pseudo-Dirac pairs are nearly degenerate in mass. This, in turn, leads to large amplitude mixing of active states as well as mixing into sterile states.

*e-mail b.mckellar@physics.unimelb.edu.au

[†]e-mail: gjs@baryon.phys.unm.edu

[‡]e-mail goldman@t5.lanl.gov

[§]e-mail mag@physics.unimelb.edu.au

1 Introduction

Conventional wisdom holds that neutrinos ought to be Majorana particles with very small masses, due to the action of a “see-saw” mechanism [1]. On the other hand, there have been recent theoretical suggestions [2, 3] that neutrinos may well be Majorana particles occurring in nearly degenerate pairs, the so-called pseudo-Dirac neutrinos. The recent studies at Super Kamiokande [4] of atmospheric neutrinos, which appear to require oscillations between nearly maximally mixed mass eigenstates, appeared to lend credence to this suggestion, although the present analyses show that this mixing cannot be entirely to sterile states [5].

There is no overriding principle to specify the structure of the mass matrix assumed for the sterile sector in flavor space. Some early discussions [6] implicitly assume that a mass term in the sterile sector should be proportional to the unit matrix. This has the pleasant prospect, in terms of the initial argument for the see-saw, that all neutrino flavors have small masses on the scale of other fermions. However, since there is no obvious requirement that Dirac masses in the neutral lepton sector are the same as Dirac masses in any other fermionic sector, this result is not compelling. Another possibility, which we shall discuss here, is that, in flavor space, the rank of the mass matrix for the sterile sector is less than the number of flavors.

In this paper, we shall concentrate on the case of a rank 1 sterile matrix, relegating the rank 2 case to some remarks at the end. Before including the effects of the sterile mass, we assume three non-degenerate Dirac neutrinos, (although this is not essential,) which are each constructed from one Weyl spinor which is active under the $SU(2)_W$ of the Standard Model (SM) and one Weyl spinor which is sterile under that interaction. Being neutrinos, both Weyl fields have no interactions under the $SU(3)_C$ or the $U(1)$ of the SM. There is then an MNS [7] matrix which relates these Dirac mass eigenstates to the flavor eigenstates in the usual manner. Note, however, that these matrix elements are not those extracted directly from experiment, as the mass matrix in the sterile sector will induce additional mixing.

We next use the Dirac mass eigenstates to define bases in both the active 3-dimensional flavor space and the 3-dimensional sterile space¹. Following the spirit of the original see-saw, we allow for no Majorana mass term in the active space.

A rank 1 sterile mass matrix may be represented as a vector of length M oriented in some direction in the 3-dimensional sterile space. If that vector lies

¹In much of the literature, the sterile space is referred to as “Right-handed” (or just R) and the active space as “Left-handed” (or just L), which follows from the behavior of the components of a Dirac neutrino where the neutrino is defined as that neutral lepton emitted along with a positively charged lepton. Since we are dealing with mass matrices, which necessarily all couple left-handed representations to right-handed representations, we choose to refer to these as active and sterile.

along one of the axes, then that Dirac neutrino will partake of the usual see-saw structure (one nearly sterile Majorana neutrino with mass approximately M and one nearly purely active neutrino with mass approximately m_D^2/M) and the other two mass eigenstates will remain Dirac neutrinos. If that vector lies in a plane perpendicular to one axis, the eigenstate associated with that axis will remain a pure Dirac neutrino, and the other two will form one pseudo-Dirac pair and a pair displaying the usual see-saw structure. Both of these pairs will be mixtures of the 4 Weyl fields associated with the two mixing Dirac neutrinos. In general, the structure will be 2 pseudo-Dirac pairs and one see-saw pair, all mixed.

As we remarked above, the very large mixing required by the atmospheric neutrino measurements was initially taken to be evidence for a scheme involving pseudo-Dirac neutrinos. (This, afterall, follows Pontecorvo's initial suggestion [8].) However, pure mixing into the sterile sector is now strongly disfavored [5]. It is evident from the discussion above that there is a region of parameter space (directions of the vector) in which the two pseudo-Dirac pairs are very nearly degenerate, giving rise to the possibility of strong mixing in the active sector coupled with strong mixing into the sterile sector. We shall explore this point in some depth.

The organization of the remainder of the paper is as follows. In the next section we present the mass matrix, discuss the parameterization of the sterile mass matrix and various limiting cases. We show the spectrum for a general case. In the section following that, we specialize to the case where the vector representing the sterile mass entry lies in a plane perpendicular to one of the axes. In this case we can carry out an analytical expansion in the small parameter $< m_D > / M$. In the fourth section we apply those results to the case where the plane in question is perpendicular to the axis for the middle Dirac mass, raising the possibility of near degeneracy between pseudo-Dirac pairs. Moving away from that plane produces large mixing amongst the members of those pseudo-Dirac pairs. Finally, we remark on the structures expected for a rank 2 sterile matrix and then reiterate our conclusions.

2 General mass matrix

The flavor basis for the active neutrinos and the pairing to sterile components defined by the (generally not diagonal) Dirac mass matrix could be used to specify the basis for the sterile neutrino mass matrix M_S . Instead we take the basis in the sterile subspace to allow the following convention. This implies a corresponding transformation of the Dirac mass matrix, which is irrelevant at present since the entries in that matrix are totally unknown.

Choose a mass eigenvalue of M and an eigenvector in the third direction. Then rotate to Dirac mass eigenstates, first by an angle of θ in the 1 – 3 plane and then by ϕ in the 1 – 2 plane. This gives a 3×3 mass matrix in the sterile

sector denoted by

$$M_S = M \begin{bmatrix} \cos^2 \phi \sin^2 \theta & \cos \phi \sin \phi \sin^2 \theta & \cos \phi \sin \theta \cos \theta \\ \cos \phi \sin \phi \sin^2 \theta & \sin^2 \phi \sin^2 \theta & \sin \phi \sin \theta \cos \theta \\ \cos \phi \sin \theta \cos \theta & \sin \phi \sin \theta \cos \theta & \cos^2 \theta \end{bmatrix}. \quad (1)$$

In this representation, the Dirac mass matrix is

$$M_D = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}. \quad (2)$$

Note that there are special cases. For $\theta = 0$ and any value for ϕ ,

$$M_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M \end{bmatrix}. \quad (3)$$

For $\theta = \pi/2$ and $\phi = 0$,

$$M_S = \begin{bmatrix} M & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (4)$$

and, for $\theta = \pi/2$ and $\phi = \pi/2$,

$$M_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

The 6×6 submatrix² of the full 12×12 is, in block form,

$$\mathcal{M} = \begin{bmatrix} 0 & M_D \\ M_D & M_S \end{bmatrix}. \quad (6)$$

Note that, in the chiral representation, the full 12×12 matrix is

$$\begin{bmatrix} 0 & \mathcal{M} \\ \mathcal{M} & 0 \end{bmatrix}. \quad (7)$$

Thus the full set of eigenvalues will be \pm the eigenvalues of \mathcal{M} . Where it matters for some analysis we keep track of the signs of the eigenvalues; however for most results we present positive mass eigenvalues.

After some algebra, we obtain the secular equation

$$\begin{aligned} 0 = & \lambda^6 - M\lambda^5 - (m_1^2 + m_2^2 + m_3^2)\lambda^4 \\ & + M[m_3^2 \sin^2 \theta + m_2^2(\sin^2 \theta \cos^2 \phi + \cos^2 \theta)]\lambda^3 \\ & + (m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2)\lambda^2 \\ & - M(m_1^2 m_2^2 \cos^2 \theta + m_2^2 m_3^2 \cos^2 \phi \sin^2 \theta \\ & + m_3^2 m_1^2 \sin^2 \phi \sin^2 \theta)\lambda \\ & - m_1^2 m_2^2 m_3^2. \end{aligned} \quad (8)$$

²We use the states rather than the field operators to define the mass matrix; see Ref.([9]).

This may be rewritten as

$$\begin{aligned}
0 = & (\lambda^2 - m_1^2)(\lambda^2 - m_2^2)(\lambda^2 - m_3^2) \\
& - \lambda M (\lambda^4 - [m_3^2 \sin^2 \theta + m_2^2 (\sin^2 \theta \cos^2 \phi + \cos^2 \theta) \\
& + m_1^2 (\sin^2 \theta \sin^2 \phi + \cos^2 \theta)] \lambda^2 \\
& + m_1^2 m_2^2 \cos^2 \theta + m_2^2 m_3^2 \sin^2 \theta \cos^2 \phi \\
& + m_3^2 m_1^2 \sin^2 \theta \sin^2 \phi).
\end{aligned} \tag{9}$$

The special cases in which the sterile mass-vector is aligned along one of the Dirac mass matrix axes follow directly. For $\theta = 0$, we find

$$(\lambda^2 - m_1^2)(\lambda^2 - m_2^2)(\lambda^2 - M\lambda - m_3^2) = 0, \tag{10}$$

for $\theta = \pi/2$ and $\phi = 0$

$$(\lambda^2 - m_2^2)(\lambda^2 - m_3^2)(\lambda^2 - M\lambda - m_1^2) = 0, \tag{11}$$

and for $\theta = \pi/2$ and $\phi = \pi/2$

$$(\lambda^2 - m_3^2)(\lambda^2 - m_1^2)(\lambda^2 - M\lambda - m_2^2) = 0. \tag{12}$$

If $m_1^2 = m_2^2 = m_3^2 = m^2$, then we find

$$(\lambda^2 - m^2)^2(\lambda^2 - M\lambda - m^2) = 0. \tag{13}$$

Note that in each of these cases the sterile neutrino and the original Dirac neutrino it is aligned with become a traditional see-saw pair.

To study the system in general, we need to pick some numerical examples. For the current exercise, we have picked the following parameters.

$$\begin{aligned}
M &= 1000 \\
m_1 &= 1 \\
m_2 &= 2 \\
m_3 &= 3.
\end{aligned}$$

For this choice, the eigenvalues have a definite pattern for all values of θ and ϕ . There are two very close pairs, with values between 1 and 3. There is one very small eigenvalue, of order 10^{-3} reflecting the ratio of m to M , and one of order 10^3 (i.e., of order M). Treating the see-saw couple as a pair allows us to present three tables of such pairs for sets of angles $\theta, \phi = \pi/8, \pi/4, 3\pi/8$.

First, for the smaller close pair

$\theta \backslash \phi$	$\pi/8$	$\pi/4$	$3\pi/8$
$\pi/8$	1.398125	1.230175	1.068477
	1.394934	1.228025	1.067688
$\pi/4$	1.809478	1.478863	1.151936
	1.808183	1.477134	1.150941
$3\pi/8$	1.877166	1.562977	1.18999
	1.876742	1.561911	1.189146

(14)

Then, for the next closest pair

$\theta \backslash \phi$	$\pi/8$	$\pi/4$	$3\pi/8$
$\pi/8$	2.038992	2.107688	2.158044
	2.038729	2.107156	2.157407
$\pi/4$	2.347974	2.46348	2.529128
	2.346047	2.462176	2.52809
$3\pi/8$	2.816525	2.847539	2.868607
	2.815691	2.846972	2.868186

(15)

Finally, even though this pair does not directly impact the argument, to present a complete set, we display the remaining pair

$\theta \backslash \phi$	$\pi/8$	$\pi/4$	$3\pi/8$
$\pi/8$	1000.008	1000.008	1000.008
	0.00444	0.005366	0.006778
$\pi/4$	1000.005	1000.006	1000.006
	0.001997	0.002717	0.004248
$3\pi/8$	1000.003	1000.003	1000.004
	0.001289	0.001819	0.003092

(16)

3 Two flavor subspace

In the next section we shall discuss the case where two of the pseudo-Dirac pairs are nearly degenerate and follow the mixing patterns as we move away from that region of parameter space. To facilitate that discussion, and to explore

a system where analytic approximations are available, it is useful to examine the limit where one Dirac mass eigenstate remains uncoupled from the other two. Anticipating the following section, we decouple m_2 . This is equivalent to examining a two flavor system in which the Dirac mass eigenvalues are m_1 and m_3 and the vector describing the sterile mass is described by $\phi = 0$.

It is useful to define some new symbols:

$$m_0^2 = m_1^2 \sin^2 \theta + m_3^2 \cos^2 \theta \quad (17)$$

$$a = \frac{(m_1^2 - m_3^2) \sin \theta \cos \theta}{m_0 \sqrt{2}} \quad (18)$$

$$b = \frac{m_1 m_3}{m_0} \quad (19)$$

and $c = \cos \theta$, $s = \sin \theta$. Note the additional $1/\sqrt{2}$ factor in a .

It helps to transform the mass matrix

$$\mathcal{M}_1 = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_3 \\ m_1 & 0 & M s^2 & M c s \\ 0 & m_3 & M c s & M c^2 \end{pmatrix} \quad (20)$$

into the form

$$\mathcal{M} = \begin{pmatrix} m_0 & 0 & 0 & a \\ 0 & -m_0 & 0 & -a \\ 0 & 0 & 0 & b \\ a & -a & b & M \end{pmatrix} \quad (21)$$

so one can see that to lowest order the three small eigenvalues are $\pm m_0, 0$. Note the minus sign on the a in the (2,4) and (4,2) positions.

The matrix effecting the transformation $\mathcal{M} = \Omega^\dagger \mathcal{M}_1 \Omega$ is

$$\Omega = m_0^{-1} \begin{pmatrix} m_1 s / \sqrt{2} & -m_1 s / \sqrt{2} & m_3 c & 0 \\ -m_3 c / \sqrt{2} & m_3 c / \sqrt{2} & m_1 s & 0 \\ m_0 s / \sqrt{2} & m_0 s / \sqrt{2} & 0 & m_0 c \\ -m_0 c / \sqrt{2} & -m_0 c / \sqrt{2} & 0 & m_0 c \end{pmatrix} \quad (22)$$

This suggests writing the characteristic equation as:

$$\mu (m_0^2 - \mu^2) \mu (M - \mu) = 2\mu^2 a^2 - (m_0^2 - \mu^2) b^2 \quad (23)$$

which is convenient for iterative solution in a series in M^{-1} . The usual equation obtained directly from $|\mathcal{M}_\infty - \mu| = 0$,

$$\mu^4 - \mu^3 M - \mu^2 (m_1^2 + m_3^2) + \mu m_0^2 M + m_1^2 m_3^2 = 0, \quad (24)$$

is just the same equation.

The solution to order M^{-2} is now

$$\mu_1 = m_0 - \frac{a^2}{M} - \frac{a^2}{m_0 M^2} \left(m_0^2 - \frac{a^2}{2} - b^2 \right) \quad (25)$$

$$\mu_2 = -m_0 - \frac{a^2}{M} + \frac{a^2}{m_0 M^2} \left(m_0^2 - \frac{a^2}{2} - b^2 \right) \quad (26)$$

$$\mu_3 = -\frac{b^2}{M} \quad (27)$$

$$\mu_4 = M + \frac{b^2}{M} + 2\frac{a^2}{M} \quad (28)$$

Notice that the eigenvalues sum to M as they must, but the $\pm m_0$ eigenvalues are shifted in the same direction at $O(M^{-1})$ and in opposite directions at $O(M^{-2})$. Note that μ_3 and μ_4 , do not pick up $O(M^{-2})$ corrections; their next correction is at the next order.

4 Nearly degenerate psudo-Dirac pairs

Applying the techniques of the last section, we find the angle θ such that $m_0 = m_2$. We then move ϕ away from 0 and display the eigenfunctions by giving the amplitudes in the original basis. To illustrate the general nature of the result, we have changed the Dirac masses from the even spacing used above.

In the following table, the Dirac masses are taken to be $m_1 = 1$, $m_2 = 1.1$, and $m_3 = 3$. This effectively means that m_1 is taken to set the scale. To display the structure of the spectrum, rather than to seek a realistic example, we have chosen $M_S = 1000$. The angles are given in degrees.

$\theta = 9.324078, \phi = 0$

mass	1active	2active	3active	1sterile	2sterile	3sterile
1.099328	0.635032	0.000000	-0.310533	0.698108	0.000000	-0.113793
1.100680	-0.633620	0.000000	0.314383	0.697413	0.000000	-0.115345
1.100000	0.000000	0.707107	0.000000	0.000000	0.707107	0.000000
1.100000	0.000000	-0.707107	0.000000	0.000000	0.707107	0.000000
0.007438	0.441883	0.000000	0.897064	-0.003287	0.000000	-0.002224
1000.008789	0.000162	0.000000	0.002960	0.162017	0.000000	0.986784

$\theta = 9.324078, \phi = 2.25$

mass	1active	2active	3active	1sterile	2sterile	3sterile
1.095953	0.479130	0.468214	-0.225940	0.525106	-0.466489	-0.082539
1.096608	0.437964	-0.514829	-0.208027	-0.480274	0.513243	0.076041
1.103359	0.416946	0.529767	-0.212981	0.460041	0.531383	-0.078333
1.104056	-0.458049	-0.484588	0.235669	0.505710	0.486376	-0.086730
0.007438	0.441553	0.015769	0.897088	-0.003285	-0.000109	-0.002224
1000.008789	0.000162	0.000007	0.002960	0.161892	0.006361	0.986784

$\theta = 9.324078, \phi = 4.5$

mass	1active	2active	3active	1sterile	2sterile	3sterile
1.092254	0.479875	-0.471453	-0.217491	0.524155	-0.468127	-0.079183
1.092888	-0.458763	0.495815	0.209390	0.501371	-0.492614	-0.076279
1.107010	0.416602	0.526536	-0.221472	0.461189	0.529886	-0.081725
1.107726	0.437718	0.503654	-0.234323	-0.484866	-0.507196	0.086521
0.007439	0.440571	0.031517	0.897156	-0.003273	-0.000217	-0.002226
1000.008789	0.000162	0.000014	0.002960	0.161517	0.012712	0.986784

$\theta = 9.324078, \phi = 22.5$

mass	1active	2active	3active	1sterile	2sterile	3sterile
1.062925	0.550356	-0.405921	-0.179528	0.584987	-0.392239	-0.063608
1.063381	0.546548	-0.411257	-0.179609	-0.581185	0.397574	0.063663
1.134871	0.337840	0.568726	-0.249457	0.383405	0.586755	-0.094367
1.135731	0.341702	0.564710	-0.254038	-0.388074	-0.583058	0.096172
0.007475	0.409265	0.154109	0.899298	-0.003058	-0.001048	-0.002241
1000.008789	0.000150	0.000068	0.002960	0.149684	0.062001	0.986784

$\theta = 9.324078, \phi = 45$

mass	1active	2active	3active	1sterile	2sterile	3sterile
1.030458	0.632073	-0.290233	-0.127244	0.651329	-0.271878	-0.043708
1.030692	-0.630801	0.292859	0.127989	0.650162	-0.274406	-0.043972
1.163620	0.226485	0.612428	-0.270973	0.263544	0.647849	-0.105102
1.164612	0.227955	0.610618	-0.274587	-0.265472	-0.646488	0.106595
0.007566	0.315141	0.286490	0.904762	-0.002384	-0.001969	-0.002282
1000.008789	0.000115	0.000126	0.002960	0.114563	0.114563	0.986784

This table represents a small part of the available parameter space and was chosen to display some possible features. First, θ was chosen so that, at $\phi = 0$, the Dirac pair at m_2 was bracketed by the pseudo-Dirac pair. Such a θ will exist for any pattern of the Dirac masses. Then, for small values of ϕ , there will be two nearly degenerate pseudo-Dirac pairs.

Note that, for $\phi = 0$, there is no mixing between the fields labelled by 2 and the remaining fields, while for the next entry at $\phi = 2.25$ degrees there is considerable mixing. That mixing continues as ϕ increases while the eigenvalues move apart. The pattern described by the centroids of the pseudo-Dirac pairs is fixed by the angles θ and ϕ . If M_S is increased, that pattern is hardly changed. The primary effect of increasing M_S , as would be expected from the analysis in the previous section, is to decrease the separation of the two members of each pseudo-Dirac pair while producing the usual see-saw behavior for the remaining pair.

The implication for oscillation phenomena is clear. A given weak interaction produces an active flavor eigenstate which is some linear combination of the three active components listed in the table. That then translates into a linear combination of the six mass eigenstates. From the table it is clear that the involvement of the heavy Majorana see-saw state is minimal, so the expansion really consists of the light Majorana see-saw state and the four Majorana states arising from the two pseudo-Dirac pairs.

Since these five mass eigenstates have both active and sterile components, the subsequent time evolution will involve both flavor change and oscillation into the sterile sector.

Finally, inspection of the column labelled 1active for $\phi = 2.25$ or $\phi = 4.5$, for example, shows that the presence of a rank 1 sterile mass matrix can seriously change any mixing pattern of the MNS type [7], from that which would have obtained with purely Dirac neutrinos.

5 Rank 2

We have not discussed the case of rank 2 matrices explicitly, although the pattern is obvious. In such a case, there would be two see-saw pairs and one pseudo-Dirac pair, leading to three active and one sterile light neutrino. While this pattern is currently being analyzed in the literature, we do not see any compelling pattern for it in the sterile sector. Furthermore, the current interpretation of the atmospheric neutrino data can be accommodated much more easily in the rank 1 case discussed in this paper. Therefore we leave the discussion of rank 2 to a longer paper.

6 Conclusions

Our analysis shows that, starting with a rank 1 mass matrix in the sterile sector, it is possible to generate in a simple way, scenarios in which one of the active neutrinos mixes in a near maximal way with both active and sterile neutrinos. The super-Kamiokande data allow such mixing.

We are encouraged by this to pursue the concepts presented here in more detail in a later paper.

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